

On Quantum Theory

Rudolf Haag

Abstract

A discussion of fundamental aspects of quantum theory is presented, stressing the essential role of “events”.¹

1 Metaphysics

Looking superficially at so many shelves with books and journals in a physics library we recognize already some basic features.

First: Physics is a collective enterprise needing exchange between many human individuals. Each person experiences sensations registered in his conscious mind. And the remarkable thing is that we can communicate with others and find a consensus about some part of such impressions. The recognition of other persons with minds independent of ours together with the possibility of communicating with them suggests that we are small parts of a huge world which we call nature or the universe, most of which we assume to be disjoint from human consciousness. The complement of conscious minds we call the outside world.

Since everything we can ever know rests ultimately in the consciousness of our minds, there is the question stressed in idealistic philosophy as to whether the assumption of an outside world is superfluous. Pursuing this we may arrive at the idea of a universal consciousness constituting the universe. Such philosophical endeavors must be understood as the search for a metaphysical model appropriate to the motivation of our actions. In the fields of fine arts, music, religion, the emphasis on the primary role of consciousness is natural, it does, however, not provide the motivation for natural sciences, particularly not for experimental physics.

¹Abstract by Erhard Seiler (see afterword)

Here we take the dualistic or realistic point of view assuming two separate realms: the outside world on the one hand, and consciousness on the other. This was the driving motivation of experimental physicists. Between the outside world and consciousness there is an interaction in both directions. On the one hand, there is the action of the outside world on us, causing impressions. On the other hand, the action from us on the outside world, producing a change there subject to our will. This is essential for physics proper, involving the planning and setting up of experiments and recognizing that the senses of touch and vision could be replaced and refined by instruments called detectors. We shall call the response of such a detector a (coarse) event. An important feature in this context is the possibility of creating documents of such events, which may be studied much later by anybody interested.

Let us make it clear that we do not imply that one or the other of these metaphysical models is true or false. We take recourse to the “*Philosophie des Als ob*” (Philosophy of “as if”) by Hans Vaihinger. (allowing any view as long as it is not contradicted...).

Second: “divisibility”. Any gain of knowledge starts with (needs) the distinction of different things: different individuals of any kind. Our ability to distinguish different elements leads us to the concept of numbers, sets, from which the whole structure of mathematics ultimately is built. Moreover, the recurrent appearance of simple parts of impressions leads us to assign symbols (words) to them, resulting in the development of language and the ability to communicate with others about our impressions. In the quasi-static case (e.g. a tree stands before my window) the symbol tree stands for a common element recognizable in an almost continuous succession of impressions, thereby integrating all of them into a single concept. We instinctively associate with the symbol tree an element of the outside world, interacting with our consciousness and causing the impression. The assumed element of the outside world responsible for such an impression is called an object. Typically it may be a piece of matter registered by our sense of touch or a ray of light interacting with our retina. The motivation for considering an object as an element of the outside world is: we can control the appearance by opening or closing our eyes, approach it with different senses, we can discuss it with our neighbors. In addition it satisfies our deeply rooted urge for causality, the demand that the impression of the tree should have a cause.

Throughout centuries of physics and chemistry the classification of stable objects, the structure of matter, has been pursued with enormous success. As the description progressed towards finer and finer

features, the visualization of objects became more and more fuzzy, so that the concept of objects no longer appears to provide a suitable basis in theory. Much closer to reality is the concept of an event, marking the result of an interaction process. In quantum physics, it marks the transition from a possibility to a fact. Its salient properties are irreversibility and an approximate localization in space and time, which provides the basis for our analysis of geometric relations in space-time.

For each person the primary phenomena (registered events) form a well-ordered sequence in time. The sequence is steadily growing. The last phenomenon in this sequence is of a special character, it is the only one felt directly. It marks the subjective present, the earlier ones are guarded in the memory or in records, the later ones are not (yet) existing. The present divides time into past and future. The past consists of unchangeable facts, the future is open.

Any statements about future events depend on inductive reasoning. Observations of regularities in the occurrence of past events have led to the formulation of laws of nature. We stipulate that these will hold in the future. Thus, they allow us to make predictions about coming phenomena.

In classical theory the laws are strictly deterministic, i.e. precise knowledge of objects and events in the past (unattainable in practice) would enable us to make precise predictions for the development in the future. Moreover it is widely believed that these laws satisfy complete symmetry between past and future (time reversal invariance); we shall return to this later.

In quantum theory past history does not allow to predict the future; the past determines only probabilities of future events. Furthermore, the observation of an event does not allow us to make retrodictions of probabilities of past events (see Appendix A). This has some bearing on the question of reversibility, as discussed in the next section.

2 Irreversibility

The irreversibility of all naturally occurring processes is a general experience. This is most strikingly evident in biological processes (the development of organisms between birth and death is obviously unidirectional). But it is also omnipresent in physics and expressed there by the second law of thermodynamics. The reconciliation of this law

with the generally assumed time reversal invariance of the basic laws of nature in classical physics is the challenge addressed by statistical mechanics. There, Boltzmann attributed to a macroscopic state a thermodynamic probability reflecting the number of different realizations of this macroscopic appearance by microscopic configurations. He interpreted this probability as the logarithm of the “entropy, whose growth with increasing time is stipulated in the second law of thermodynamics. These probabilities differ enormously between various macroscopic states and it is plausible to expect that under many circumstances the macroscopic body will change its state to one with higher thermodynamic probability in the passage of time.

This is for instance so in laboratory physics, where the experimentalist sets the initial time at which a situation far from equilibrium (i.e. with low probability) is artificially created. It is also illustrated by dissipative effects, where typically a simple physical system describable by a few degrees of freedom begins to share energy with many untraceable degrees of freedom. Boltzmanns argument by itself, however, does not suffice to justify the second law of thermodynamics. The differences in thermodynamic probability between various macroscopic states by themselves do not logically imply any distinction between past and future. We have to put in the additional information that at the beginning we start with a state of low probability.

Irrespective of this, our above remarks about events in quantum physics point to a more elementary justification of irreversibility. We started with phenomena directly registered in our conscious mind and observed their sequential character leading to the psychological arrow of time. We then related these mental impressions to events. We shall show in the next section that the growth of the entropy is directly connected to the occurrence of events. Therefore, the thermodynamic arrow of time is coupled to the psychological arrow. We can adapt this argument to the case of classical physics. The notion of event must there be replaced by that of a process in which originally independent and distinguishable objects begin to interact, wiping out their distinguishability.

We sometimes wonder why the dogma of time reversal invariance of the fundamental laws is so firmly entrenched, since it is not always clear what should be regarded as a fundamental law. In classical physics there is at least one aspect of dissipation that is of a fundamental nature. This is the radiation damping associated with any acceleration of a charged particle. In spite of Diracs beautiful analysis [1] and many subsequent elaborations such as [2]) no formulation free from objections exists. Thus, strictly speaking, no complete

self-consistent classical theory uniting mechanics and electrodynamics exists.

In Quantum Physics this is related to the infra-red problem and leads to the observation that the causal shadow of each event is its forward light cone. This leads to a formulation of the dynamics in terms of a semi-group shifting light cones inside each other, as worked out by Buchholz and Roberts [3].

The strongest argument for the belief in time reversal invariance is the CPT theorem [footnote: T stands for Time reversal, C for Charge conjugation, P for space reflection], which states that in any quantum field theory satisfying the standard axioms there exists a CPT operator, implying that the equations of motion are CPT invariant. But, these equations of motion do not tell us what actually happens. They only describe the development of the so-called quantum state, which determines the probability for the realization of different possible events. To arrive at facts (individual events) we must add to these equations the statistical interpretation of the quantum state, which in our context we might call the principle of random realization ([4]).

It is this principle which destroys time reversal invariance and unites the various arrows of time discussed in the literature. To repeat the main line of our reasoning: we started with phenomena directly registered in our conscious mind and observed their sequential character leading to the psychological arrow of time. We then related these mental impressions to events, namely interaction processes which may be attributed to an outside world, and transported the sequential character from the phenomena to the events. Irreversibility happens in the transition from a possibility to a fact, which in quantum physics is governed by the principle of random realization, whereas in classical physics the state itself is supposed to describe reality and its deterministic propagation leaves no room for a distinction between possibilities and facts.

3 Standard Questions and Procedure

3.1 Observables

One decisive difference between the classical and quantum descriptions of nature is seen in the following statement of Niels Bohr: “We cannot assign any conventional attributes to an atomic object”. As “conventional” we may regard an attribute describable numerically.

Of course there are defining properties of an object, such as mass and charge of an electron; but beyond this in classical physics we imagine that an object may appear in variable states, for instance that a particle has a definite position at any given time. Bohr's statement negates this.

Instead of attributes of an object one uses the concept of observables. This corresponds to the idea that one can carry through various measurements on an object, each of them leading to a numerical value, the observational result. The difference is that before the measurement no definite numerical value is assignable to the object. This value arises only by the interaction between the object and the measuring device. Every single observed value has to be considered a fact. As such it is irreversible, usually approximately localized in space and time; thus it corresponds to what we called an event above.

As a simple example of an object let us consider an electron. One possible observable is its position at a given time. It may be observed as a dot appearing at that time on a scintillation screen. This cannot be regarded as the position of the electron before the measurement, but is created by the interaction between the electron and the screen. The coordinates of this dot are not attributes of the electron but attributes of the interaction event.

A more interesting example is to choose as observable the internal energy of an atom. The possible measuring values are the energy levels of the electrons in the atom. These will manifest themselves by spectral lines corresponding to frequencies of the emitted light, which are proportional to energy differences. These frequencies in turn may be determined for instance by letting the light pass through a diffraction grating and recording the outcome on a photographic plate. Thus the observation of energy levels ultimately again leads to a position measurement. We may note that in the last resort observations always terminate in a position measurement, since they record the location of an event.

The theoretical description uses the mathematics of Hilbert space. An observable is represented by a self-adjoint operator. These operators have a spectral resolution, i.e. spectral values and spectral projectors; the former are interpreted as possible measuring values.

The second characteristic feature of quantum physics is its lack of determinism. It only yields statistical predictions for the outcome of observations. These are governed by the so-called quantum state which is mathematically represented by a positive operator ρ of unit trace, called the statistical operator. The probability of finding a

measuring value in an interval $[a, b]$ is given by $\text{tr}(\rho P_{[a,b]})$, where $P_{[a,b]}$ denotes the spectral projector for the interval $[a, b]$ and ρ describes the quantum state before observation.

Since the theory yields only statistical predictions we need for checking them an ensemble of copies of equally prepared objects and we estimate probabilities by relative frequencies of occurrence. The realistic ensemble used in this test is idealized to a Gibbsian ensemble of infinitely many independent copies.

3.2 Geometry of quantum states; composition and decomposition of systems; entanglement

On the mathematical side the set of positive operators with unit trace is a convex set which means that for any two such operators ρ_1, ρ_2 all the convex combinations $\lambda\rho_1 + (1 - \lambda)\rho_2$ with $0 \leq \lambda \leq 1$ belong again to this set. This corresponds to the possibility of mixing the two ensembles with weights $\lambda, 1 - \lambda$, respectively.

The convex body of states has extremal points, the “pure states” which cannot be written as convex combinations of others. They are represented by one-dimensional projectors or equivalently by the rays in Hilbert space on which these project. In Quantum Mechanics of particles the wave function describes such a ray. The salient feature of quantum physics is that the convex body of states is not a simplex. Thus, while an arbitrary state can be written as a convex combination of pure states, this decomposition is highly nonunique. In physical terms the decomposition of a state into a convex combination corresponds to a decomposition of the ensemble into subensembles. Therefore it is often not possible or meaningful to assume that each individual system is in some pure state. The assignment of a particular pure state to an individual system means only that this system is filed as a member of a particular subensemble whose choice remains to some extent arbitrary.

This nonuniqueness has been the source of long disputes beginning with the EPR “paradoxon” [6], the concept of entanglement between states introduced by Schrödinger [7], the inequalities by Bell [8] and Clauser et al [9] and their subsequent experimental study by Aspect [10, 11] and many later experiments.

For this discussion we must consider general systems composed of several subsystems. The Hilbert space associated with such a system is taken to be the tensor product of the Hilbert spaces of the subsys-

tems which compose it. Its set of quantum states contains “product states” $\rho^{(1)} \otimes \rho^{(2)} \otimes \dots$ and convex combinations thereof, called separable states, which describe correlations well known in classical statistics. These types of states do not, however, exhaust all possibilities. For instance, the pure states of the compound system are by definition not convex combinations of any other states, hence they cannot be separable unless they are just product states. States which are not separable are called “entangled”. The statistical predictions for such entangled states cannot be described in terms of correlations between individual states of the subsystems. A simple illustration of this feature is afforded by a thought experiment suggested by Bohm [12] and analysed in [8] and [9]:

A spin-0 particle decays into two spin-1/2 particles moving in opposite directions for a long time till one of them enters the lab of Alice, the other one the lab of Bob. In both cases the arriving particle is subjected to a measurement of the spin orientation by a Stern-Gerlach arrangement. This can yield two possible outcomes: parallel or antiparallel to the orientation of the Stern-Gerlach magnet. We denote this result by (\mathbf{a}, α) , where \mathbf{a} is the unit vector describing the direction of the magnet; $\alpha = \pm 1$ distinguishes the two possible results. The spin-part of the two-particle wave-function after the decay is a singlet state and this will remain so practically unchanged up to the detection process. This singlet state is a pure entangled two-particle state and one can show that it is impossible to assign any “conventional attributes” (“hidden variables”) nor even a quantum state to the individual particles. The former impossibility has been demonstrated in [8], the latter in [9]. We shall follow here the arguments of [9], as presented in [13].

The ensemble of all particles received by Bob may be described by an impure one-particle quantum state ρ_B . Since the twin particles are correlated due to their common birth it is not surprising that the probability for a particular measuring results of Bob is correlated with the result of Alice’s measurement on the twin. However, entanglement is more than ordinary correlation.

Suppose now that a particle is endowed with some objective property λ (which may be a quantum state or a conventional attribute) and the joint probability in the ensemble of pairs of particles is given by a distribution function $f(\lambda_1, \lambda_2)$ which describes ordinary correlation between λ_1 and λ_2 . We assume further that λ determines the probability $w(\lambda; \mathbf{a}, \alpha)$ for the measuring result (\mathbf{a}, α) , yielding the expectation value conditioned on λ

$$\langle \mathbf{a}; \lambda \rangle = w(\lambda; \mathbf{a}, +) - w(\lambda; \mathbf{a}, -). \quad (1)$$

We note that $w(\lambda; \mathbf{a}, +) + w(\lambda; \mathbf{a}, -) = 1$ because in the measurement of \mathbf{a} , one of the alternatives ± 1 must occur. The joint probability for $(\mathbf{a}, \alpha; \mathbf{b}, \beta)$ is then

$$W(\mathbf{a}, \alpha; \mathbf{b}, \beta) = \int d\lambda_1 d\lambda_2 f(\lambda_1, \lambda_2) w(\lambda_1; \mathbf{a}, \alpha) w(\lambda_2; \mathbf{b}, \beta). \quad (2)$$

For the expectation value in the joint measurement, which is defined by

$$\langle \mathbf{a}; \mathbf{b} \rangle \equiv W(\mathbf{a}, +; \mathbf{b}, +) - W(\mathbf{a}, +; \mathbf{b}, -) - W(\mathbf{a}, -; \mathbf{b}, +) + W(\mathbf{a}, -; \mathbf{b}, -) \quad (3)$$

one obtains the representation

$$\langle \mathbf{a}; \mathbf{b} \rangle = \int d\lambda_1 d\lambda_2 f(\lambda_1, \lambda_2) \langle \mathbf{a}; \lambda_1 \rangle \langle \mathbf{b}; \lambda_2 \rangle. \quad (4)$$

From this together with the positivity and normalization of the distribution function $f(\lambda_1, \lambda_2)$ one obtains inequalities between expectation values for combinations of measurements with different orientations of the apparatuses,

$$|\langle \mathbf{a}; \mathbf{b} \rangle + \langle \mathbf{a}; \mathbf{b}' \rangle| + |\langle \mathbf{a}'; \mathbf{b} \rangle - \langle \mathbf{a}'; \mathbf{b}' \rangle| \leq 2. \quad (5)$$

The experimentally observed violation of this inequality shows that the assumption of an ordinary correlation between assumed properties λ_1, λ_2 is not tenable. Instead one has the following situation: if Bob receives the information from Alice about what she has done and found in her measurements, he can split his ensemble into two subensembles according to Alice's measuring result $\alpha = \pm 1$ on the twin. Then these subensembles define two orthogonal pure states which depend on the orientation of Alice's device. It must be stressed that this has nothing to do with any physical effect of Alice's measurement on the particles received by Bob. Nor is it important how fast the information is transmitted. Bob and Alice can get together leisurely after the experiments are finished to evaluate their records. They only have to establish the correct pairing of the events, which can be found for example from the records of the arrival times. No witchcraft is involved. It shows, however, that the pure state of the particle has no objective significance. It does not describe a property of the individual particle but only the defining information about the subensemble in which the particle is filed. And here this is determined by the result of Alice's measurement on the twin.

This implies an enhancement of Bohr's tenet mentioned in the introduction. Not only can we "not assign any conventional attribute to

an atomic object” but we cannot even assign any individual quantum state to the particle. It puts in question the traditional picture of the reality of “atomic object” (particles). Nicholas Maxwell has coined the term “Propensiton” for such an object [14]. It propagates according to a deterministic law such as the Schrödinger equation which is invariant under time reversal. But it does not represent any phenomenon. It is the carrier of propensity contributing to probability assignments.

A Temporal asymmetry of quantum probabilities

For simplicity we consider here only two ‘observables’ A, B given by self-adjoint operators with simple discrete spectrum. Assume that in a state ρ first A , then B is measured. Let p_α be the probability that the measurement of A yields the spectral value α , $p_{\alpha\beta}$ the probability that the consecutive measurement of B then yields β . We have, according to the Lüders rule [5]

$$p_\alpha = \text{tr}(P_\alpha \rho), \quad p_{\alpha\beta} = \text{tr}(Q_\beta P_\alpha \rho P_\alpha Q_\beta). \quad (6)$$

Using the assumptions about the spectra we have

$$P_\alpha \rho P_\alpha = P_\alpha \text{tr}(\rho P_\alpha) \quad (7)$$

and hence, using the cyclicity of the trace

$$p_{\alpha\beta} = \text{tr}(P_\alpha Q_\beta) \text{tr}(P_\alpha \rho) = |\langle \alpha | \beta \rangle|^2 \langle \alpha | \rho | \alpha \rangle = p_\alpha |\langle \alpha | \beta \rangle|^2. \quad (8)$$

The last formula shows already the asymmetry between the two measurements.

More explicitly, when α has been measured, the probability of finding β afterwards is known and given by that formula; if, on the other hand, we only know that β has been measured in the second step, the probability that the first measurement yielded α cannot be inferred unless $|\alpha\rangle = |\beta\rangle$ (in which case it is 1).

References

- [1] P. A. M. Dirac (1938), “Classical theory of radiating electrons”, Proc. Roy. Soc. A **167**: 148–169.
- [2] R. Haag (1955), “Die Selbstwechselwirkung des Elektrons”, Z. Naturforschg. **10**: 752–761.

- [3] D. Buchholz and J. E. Roberts (2014), “New Light on Infrared Problems: Sectors, Statistics, Symmetries and Spectrum,” *Commun. Math. Phys.* **330**: 935–972 [arXiv:1304.2794 [math-ph]].
- [4] R. Haag (2013), “On the Sharpness of Localization of Individual Events in Space and Time,” *Found. Phys.* **43** 1295–1313; arXiv:1303.6431 [quant-ph].
- [5] G. Lüders (1951), “Über die Zustandsänderung durch den Meßprozess”, *Annalen d. Physik* **8**: 663–670.
- [6] A. Einstein, B. Podolsky, N. Rosen (1935). “Can Quantum-Mechanical Description of Physical Reality be Considered Complete?”, *Physical Review* **47** (10): 777-780.
- [7] E. Schrödinger, M. Born. (1935). “Discussion of probability relations between separated systems”. *Mathematical Proceedings of the Cambridge Philosophical Society* 31 (4): 555-563
- [8] J. S. Bell (1964). “On the Einstein Podolsky Rosen Paradox”, *Physics* 1 (3) (1964): 195-200.
- [9] J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt (1969), “Proposed experiment to test local hidden-variable theories”, *Phys. Rev. Lett.* 23 (15): 880–884.
- [10] A. Aspect, P. Grangier and G. Roger (1982), “Experimental realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New violation of Bell’s inequalities,” *Phys. Rev. Lett.* **49**: 91–97.
- [11] A. Aspect, J. Dalibard and G. Roger (1982), ‘Experimental test of Bell’s inequalities using time varying analyzers”, *Phys. Rev. Lett.* **49**: 1804–1807.
- [12] D. Bohm (1951), “Quantum Theory”. Prentice-Hall, Inc., Englewood Cliffs, New Jersey , p. 614.
- [13] J. S. Bell (1987). “Speakable and unspeakable in quantum mechanics”, Cambridge University Press, Cambridge, p. 37.
- [14] N. Maxwell (2011), “Is the quantum world composed of propensitons?” In: M. Suarez (ed.), “Probabilities, Causes and Propensities in Physics”, pp. 221–243, Springer, Dordrecht.

Afterword²

Rudolf Haag worked on this manuscript since the spring of 2013 until just a few weeks before his death on January 5th, 2016. It evolved in numerous discussions with me as well as Heide Narnhofer and Berge Englert. Technical help was also provided by Albert Haag and Friedrich Haag. But I should stress that the thoughts and the wording are entirely due to Rudolf Haag.

Obviously the paper has not been completed; among the further issues that Rudolf wanted to address were

- Partition of the universe, classification and reality of objects, reductionism and its possible limits;
- Indistinguishability of particles; distinguishability of particles as connections between events;
- Space and time, symmetries and invariances.

²by Erhard Seiler (Max-Planck-Institut für Physik, München, Germany, e-mail: ehs@mpp.mpg.de)